Sunday Risk Measures

Stephen J. Mildenhall

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ST. JOHN'S UNIVERSITY
Tobin College of Business
School of Risk Management

Challenges for ERM and Insurance Operations...

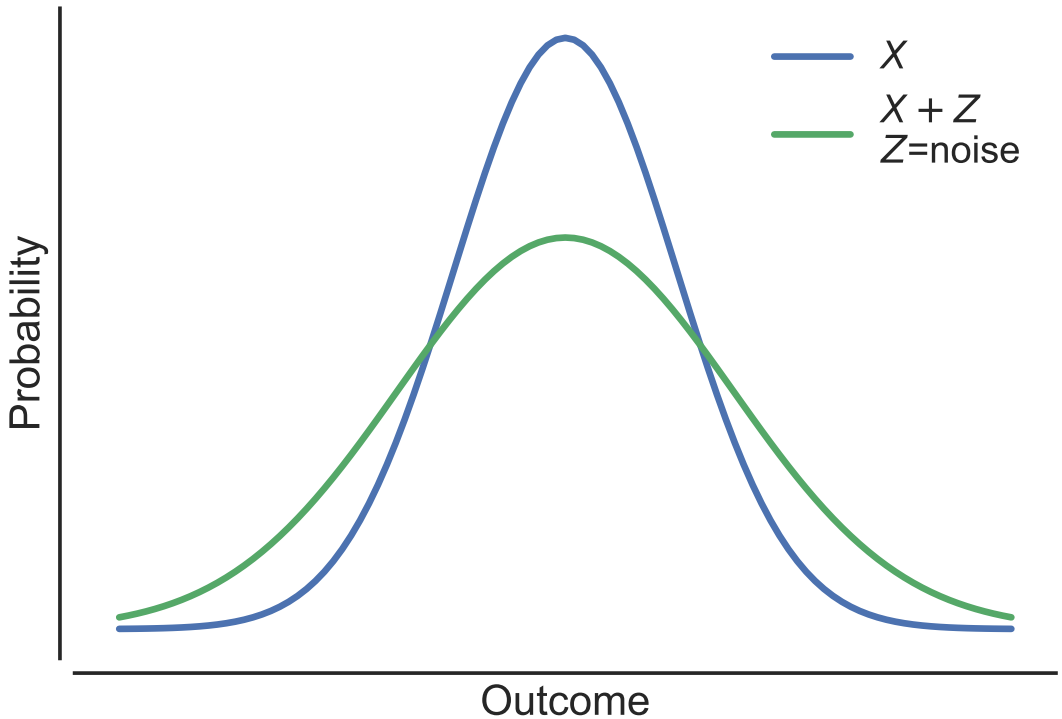
...all require **risk** quantification

- Pricing and underwriting
- Capitalization and capital cost allocation
- Portfolio optimization
- Risk adjusted LOB performance
- Reinsurance value assessment
- Reinsurance cost allocation

...but just what is *risk*??

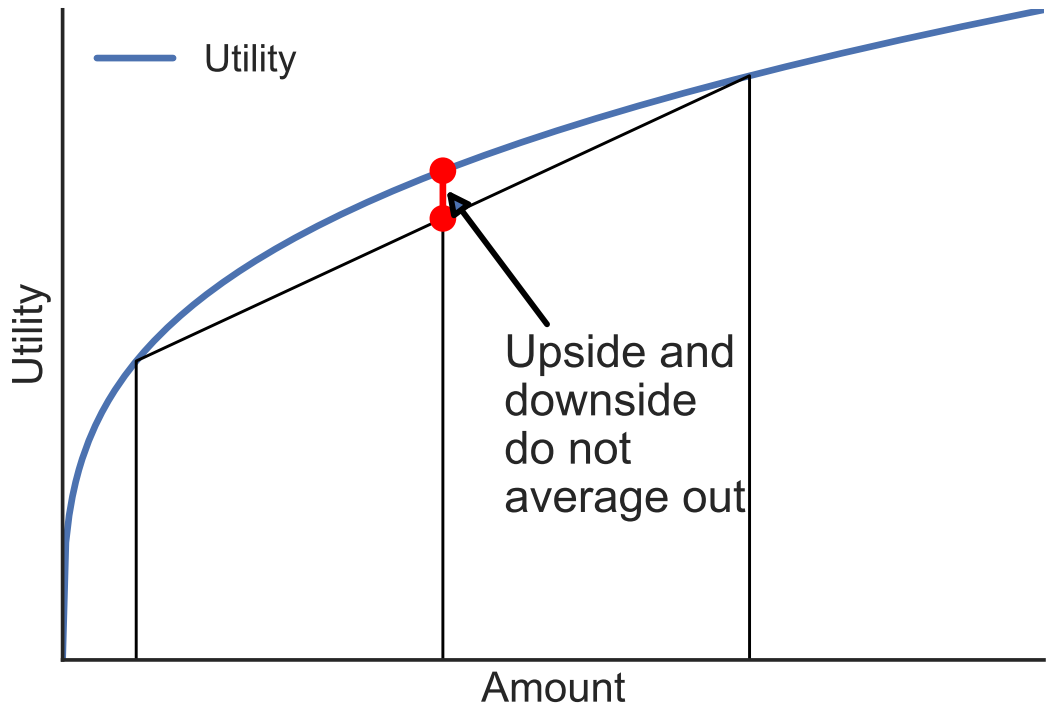
Definition I: Y is More Risky Than X if...

Y equals X plus a mean zero *noise* random variable



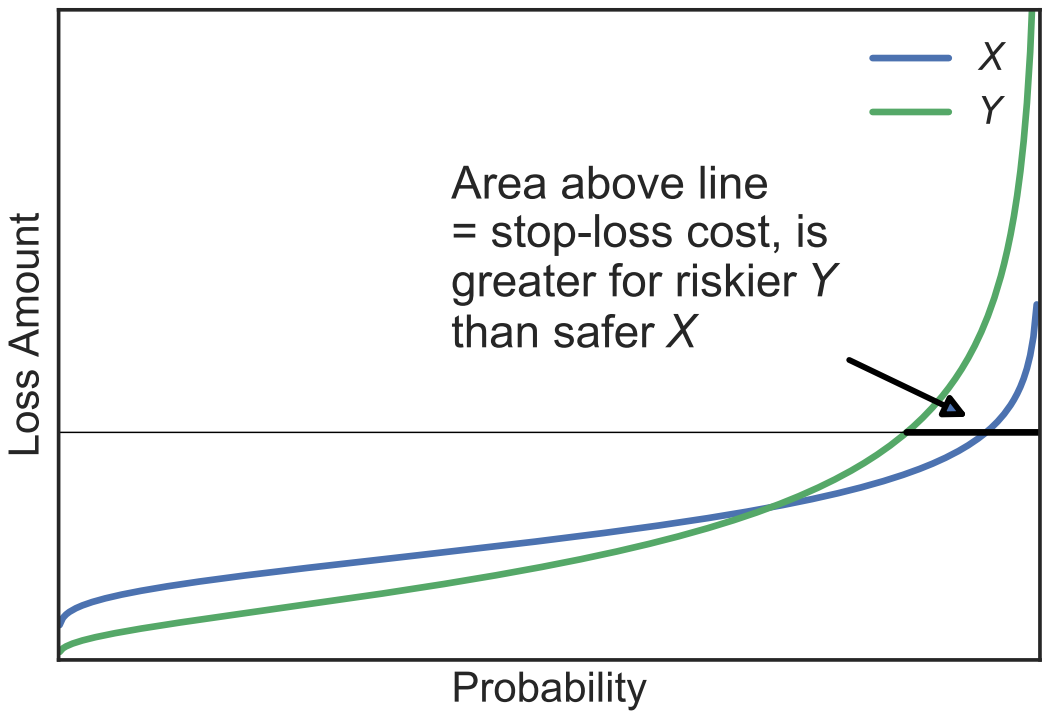
Definition II: Y is More Risky Than X if...

X and Y have the same mean and every person who prefers more to less but has decreasing marginal utility prefers X to Y



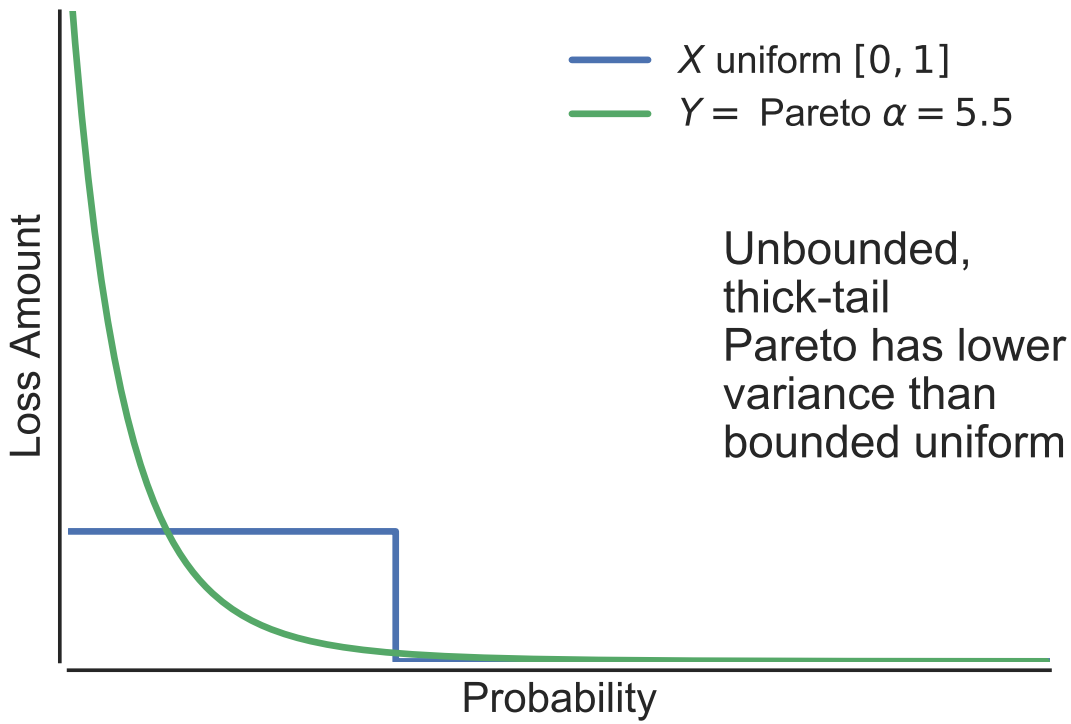
Definition III: Y is More Risky Than X if...

X and Y have the same mean and Y has more weight in the tails than X = stop-loss insurance on Y is more expensive than on X



Definition IV: Y is More Risky Than X if...

Y has greater variance than X



Definition of Risk

(Noise $\Leftrightarrow$ Utility $\Leftrightarrow$ Stop-Loss) $\not\Longleftrightarrow$ Variance

- Defining and measuring risk is **difficult**
- Need heuristics. . . called risk measures

Risk Measures Express Risk Preferences

- A **risk measure** ρ is a function associating a number $\rho(X)$ to a random variable X so that a variable X is **preferred** to $Y \Leftrightarrow \rho(X) \leq \rho(Y)$

Examples of Risk Measures

- Standard deviation, variance
- Semi-variance, one-sided
- Value at risk
- Tail value at risk
- Expected policyholder deficit
- Esscher transform
- Minimum entropy risk
- RBC, BCAR, Solvency II
- Scenario loss, Lloyd's RDS
- ...

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- Appropriate risk measure varies with **intended purpose** and must have properties consistent with economic theory

Relevant Economic and Financial Theories

Utility theory

- We all prefer more of a good thing to less
- Many exhibit **diminishing marginal utility** (DMU)
- DMU $\Rightarrow$ downward sloping demand curve
- Utility function U adjusts outcome wealth x to $U(x)$
- Utility theory: DMU $\Leftrightarrow$ risk averse, the definition!
- Utility **relative** to current wealth
- Confounds attitudes to wealth and risk

Relevant Economic and Financial Theories

Dual utility theory

- Disliking risk $\not\Longleftrightarrow$ DMU
- Corporations not DMU but may still dislike risk
- **Dual Utility**: adjust probabilities not outcomes
- Spectral risk measures adjust probabilities
- Google **Yaari** for more details!

Relevant Economic and Financial Theories

General equilibrium models

- Prices determined by supply and demand for a broad range of contracts
- Equilibrium prices equalize marginal utilities
  - equilibrium solution = quota share aggregate output
  - resulting pooling diversifies all diversifiable risk
  - consumption proportional to aggregate production and
  - consumption inversely related to individual risk aversion

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- The price today of a contract for a dollar in a future **worse** (lower aggregate production) state **must** be **greater than** that for a dollar in a future good state
  - Consumption lower implies marginal utility = state price higher
  - Adjusted probabilities must reflect state price of a dollar

Relevant Economic and Financial Theories

No arbitrage

- Equilibrium prices cannot contain opportunities for arbitrage = riskless gain
  - Not equilibrium if you prefer more to less!
- Prices have no arbitrage $\Leftrightarrow$ they are consistent with a set of event probabilities
  - Positive, additive state price density
  - Static, point-in-time, conditions: easy
  - Dynamic, over-time, conditions: harder, using stochastic processes
  - Equivalent martingale (no trend) measure

Relevant Economic and Financial Theories

No arbitrage can determine general equilibrium solution

- If a risk process has a **unique** set of allowable probabilities they **must be** the price process
  - Black-Scholes
  - Representation theorem: hedging portfolio

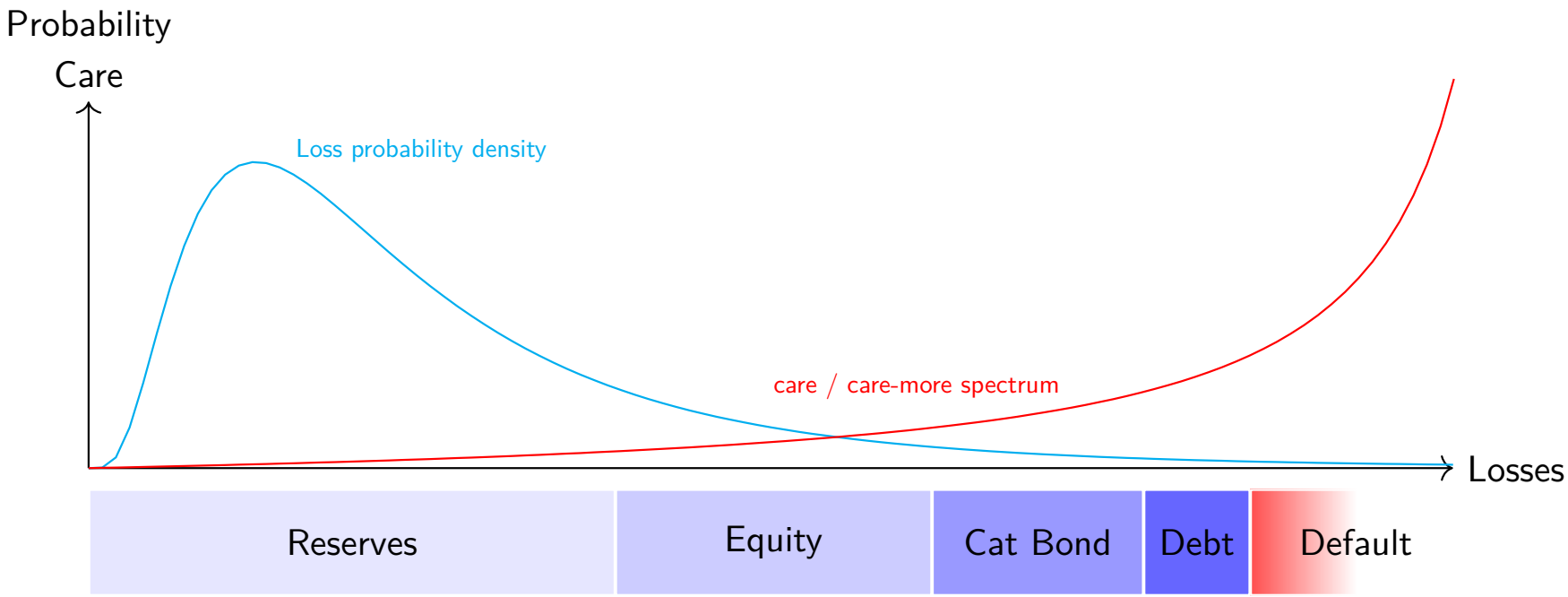
Relevant Economic and Financial Theories

Connections between general equilibrium and no arbitrage

- General equilibrium prices $\Rightarrow$ no arbitrage, but
- No arbitrage $\not\Rightarrow$ general equilibrium

Implications For Insurance and Spectral Risk Measures

- However much I care about a loss of \$10 million I **must care more** about a loss of \$11 million
- Whenever a loss of \$11 million occurs a loss of \$10 million has also occurred



Implications For Insurance and Spectral Risk Measures

If the care/care-more curve...

- does not **integrate to 1** the associated risk measure will assign **net risk to a certain payment**
- is not **positive** the associated risk measure will **price a lower risk higher**
- is not **increasing** the associated risk measure will **not respect diversification**
- On Tuesday and Wednesday we will see explicitly how to build a risk measure from a care/care-more spectrum
- Jesse will now look at various examples